CUHK Croucher Course 2026

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arXiv: 2605.29113, 2605.28967, 2605.28925

Crash course on the Lindblad master equation

$$\mathscr{L}[\rho] = \dot{\rho} = -i[H,\rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

coherent part
(unimportant)

jump operators: coupling
to environment

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steady state(s): $\mathscr{L}[\rho_{ss}] = \dot{\rho}_{ss} = 0$ $\quad e^{t\mathscr{L}}[\rho_0] \to \rho_{ss}$

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steady state(s): $\mathscr{L}[\rho_{ss}] = \dot{\rho}_{ss} = 0$ $\qquad e^{t\mathscr{L}}[\rho_0] \to \rho_{ss}$

mixing time : $\min\{t \geq 0 : \|\rho(t) - \rho_{ss}\|_1 \leq \epsilon, \forall \rho(0)\}$

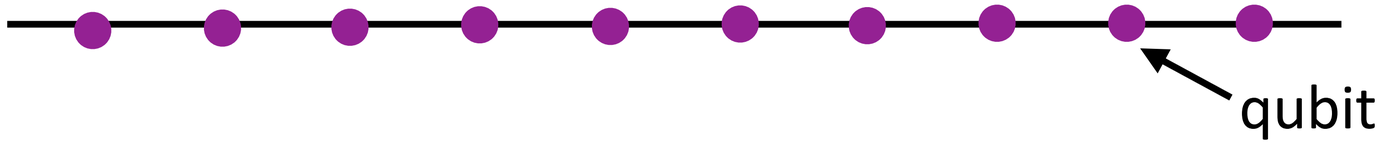
fast : $t_{\text{mix}} \sim \mathcal{O}(\log L)$

slow : $t_{\text{mix}} \sim \mathcal{O}(\text{poly}(L))$ $\qquad$ $\exp(L)$: long lived metastable state

Strong symmetries

Global symmetry operators $\{U_g\}$

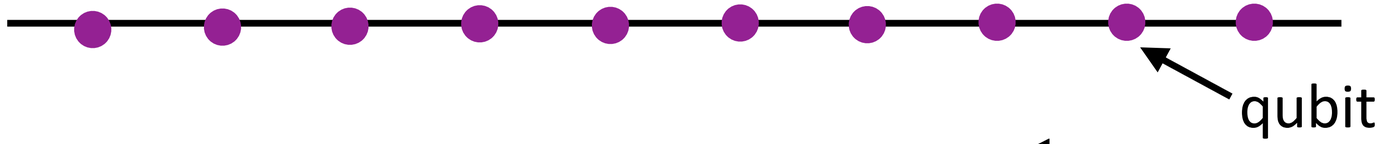
this talk: $G = \mathbb{Z}_2, U = \prod_r X_r$



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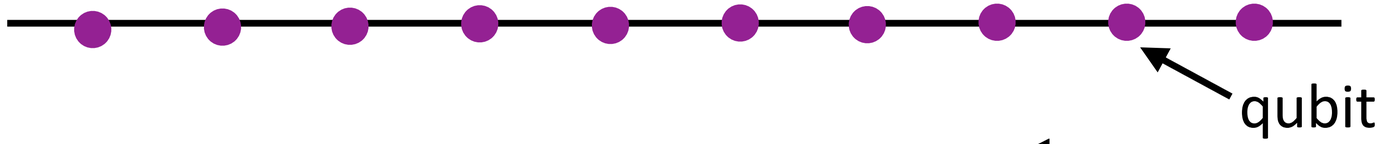
$$\mathscr{L}[\rho] = \dot{\rho} = -i[H,\rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

$\mathscr{L}$ strongly symmetric: $[H, U] = [L_k, U] = 0 \qquad \forall k$

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$\mathscr{L}$ strongly symmetric: $[H, U] = [L_k, U] = 0 \qquad \forall k$

$\exists \rho_{ss}$ in each symmetry sector : $U e^{t\mathscr{L}}[\rho] = e^{t\mathscr{L}}[U\rho] = \pm\, e^{t\mathscr{L}}[\rho]$

Warm-up: 0+1d

Equilibrium

$$H = \frac{1 - X}{2}$$

$$|\mathrm{GS}\rangle = |+\rangle \qquad E = 0$$

$$|\mathrm{EX}\rangle = |-\rangle \qquad E = 1$$

Warm-up: 0+1d

Equilibrium

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Open

$$\mathscr{L}^0[\rho] = \frac{1+X}{2}\rho\frac{1+X}{2} + Z\frac{1-X}{2}\rho\frac{1-X}{2}Z - \rho$$

$$\rho_{ss} = |+\rangle\langle+| \qquad E = 0$$

$$\rho_{ex} = |-\rangle\langle-| - |+\rangle\langle+| \qquad E = -1$$

Warm-up: 0+1d

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$\rho_{ss} = |+\rangle\langle+| \qquad E = 0$

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$e^{t\mathscr{L}}[\rho_{ss} + \alpha\rho_{ex}] = \rho_{ss} + \alpha e^{-t}\rho_{ex} \rightarrow$ steer toward ρ_{ss}

unique ss: explicitly break strong X symmetry

Phases with strong Z2 symmetry in 1+1d

$\rho \propto (1 + U)$

Strong-to-weak SSB

$\rho = |+\rangle\langle+|$

Strongly symmetric

$\rho = |\mathrm{GHZ}+\rangle\langle\mathrm{GHZ}+|$

Strong-to-trivial SSB

Phases with strong Z2 symmetry: SWSSB

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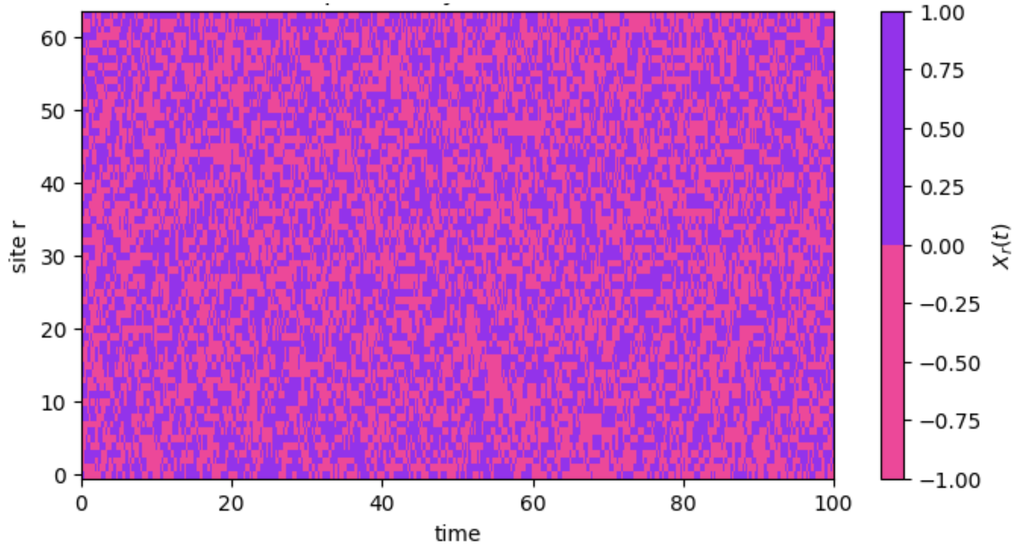
SWSSB : fast, symmetric parent Lindbladian

$$\mathscr{L}_r^{\mathrm{SWSSB}}[\rho] = X_r \rho X_r + Z_{r-1} Z_r \rho Z_{r-1} Z_r - 2\rho$$

$\downarrow$ diag in X basis

$(++) \leftrightarrow (--)$ $\quad$ $(+-) \leftrightarrow (-+)$

$(r-1, r)$



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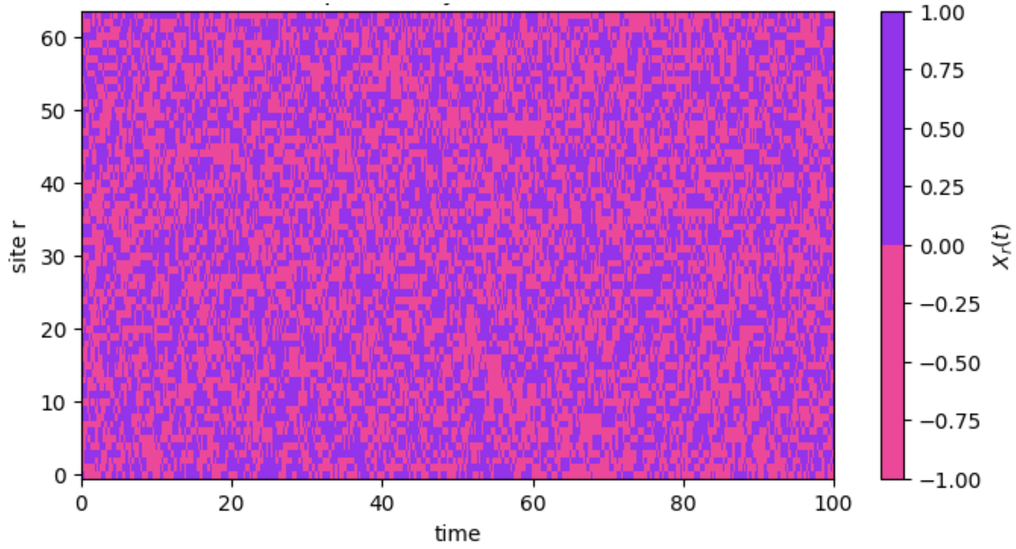
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$$\rho_- \propto (1-U)$$



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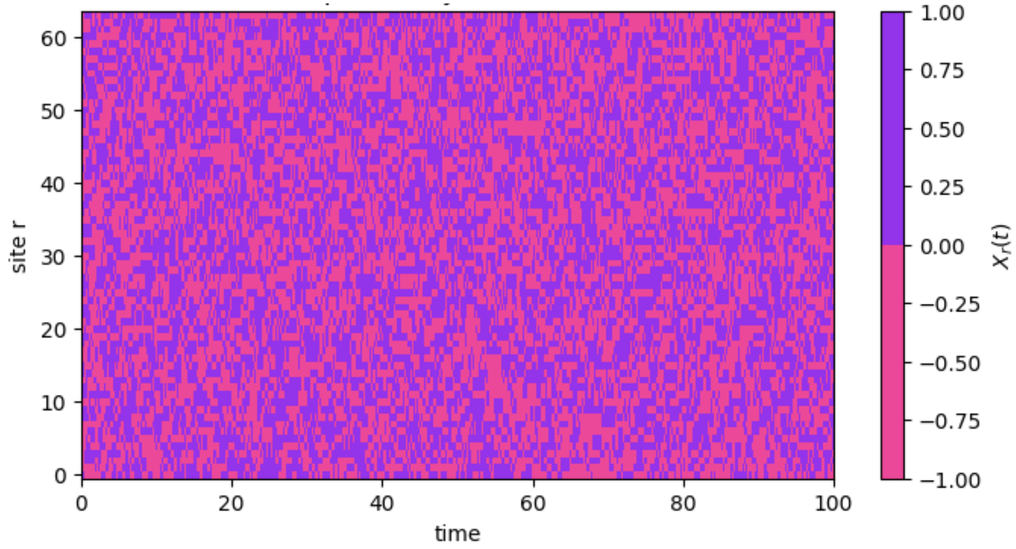
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Can similarly construct fast symmetric Lindbladian for SWSSB of anomalous $\mathbb{Z}_2$

Phases with strong Z2 symmetry: symmetric

$\rho \propto (1+U)$

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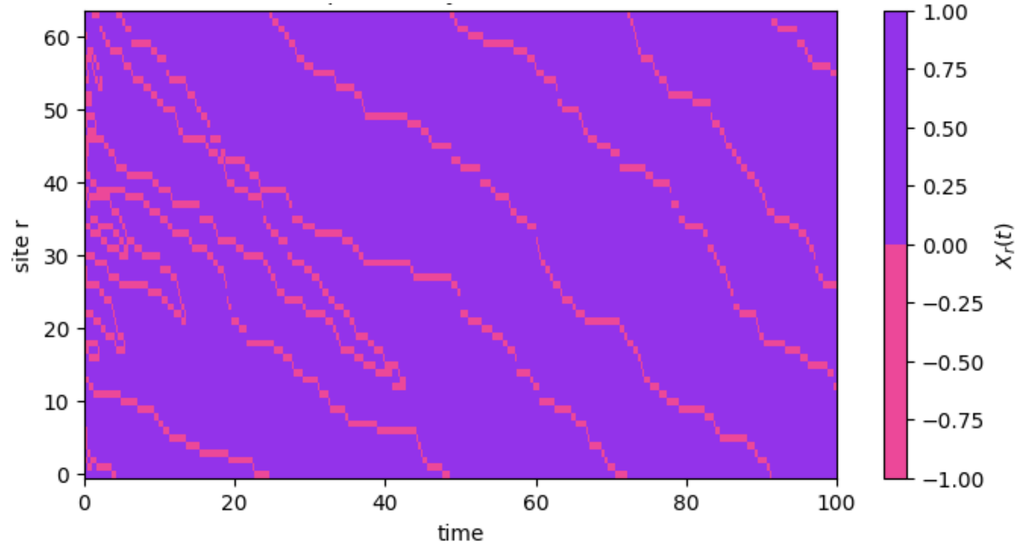
Strongly symmetric : fast, symmetry breaking or slow, symmetric parent Lindbladian

$$\mathscr{L}_r^0[\rho] = \frac{(1+X_r)}{2}\rho\frac{(1+X_r)}{2} + Z_r\frac{(1-X_r)}{2}\rho\frac{(1-X_r)}{2}Z_r - \rho$$

↓ diag in X basis

$$(+),(-) \to (+)$$

no ρ_-



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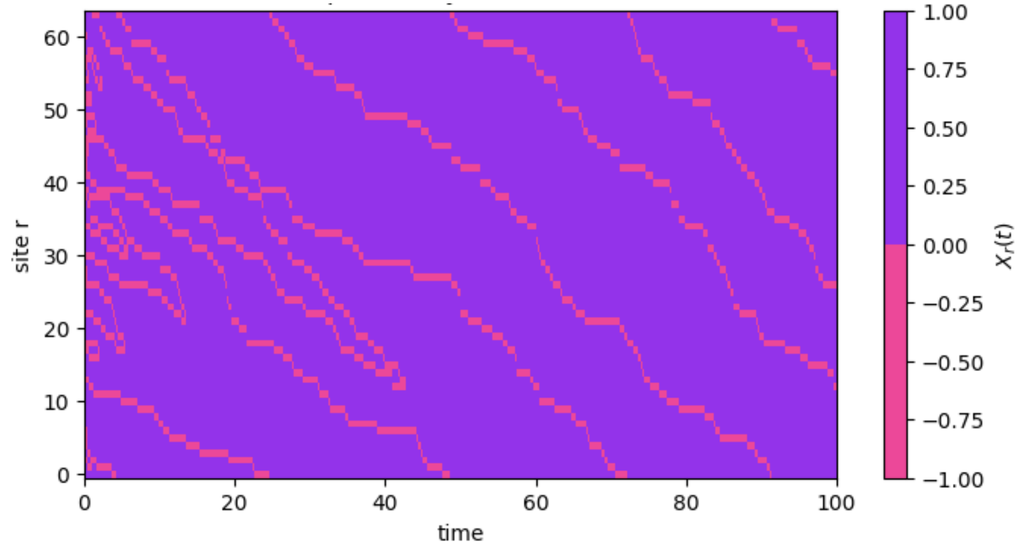
$$\mathscr{L}_r^+[\rho] = \frac{(1+X_r)}{2}\rho\frac{(1+X_r)}{2} + Z_{r-1}Z_r\frac{(1-X_r)}{2}\rho\frac{(1-X_r)}{2}Z_{r-1}Z_r - \rho$$

↓ diag in X basis

$$(++),(--)\rightarrow(++)$$
$$(-+),(+-)\rightarrow(-+)$$

$$\rho_- = \frac{1}{L}\sum_r Z_r|+\rangle\langle+|Z_r$$

(long range correlations!)



Phases with strong Z2 symmetry: symmetric

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Strongly symmetric

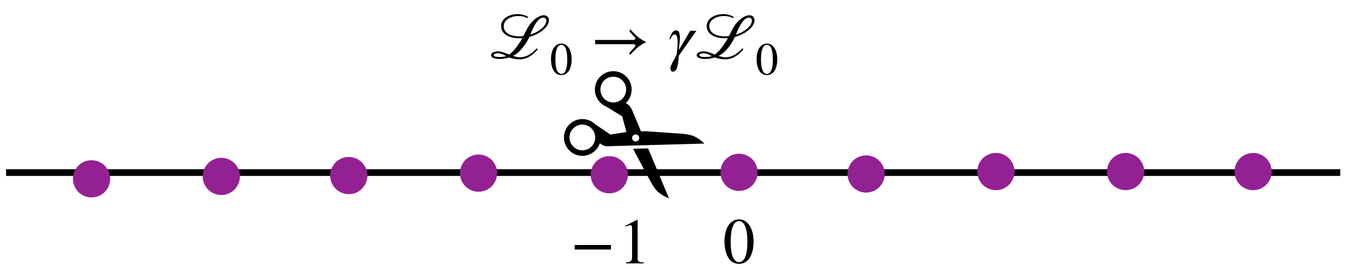


Parent Lindbladian doesn't need to be gapless! But has to be slow.

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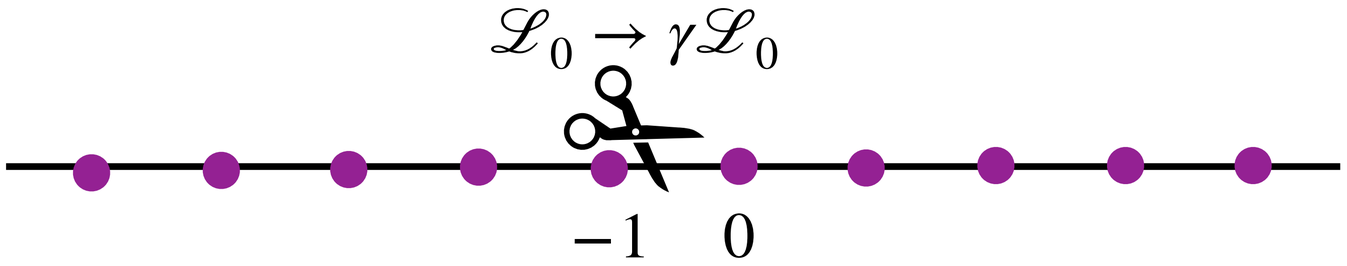


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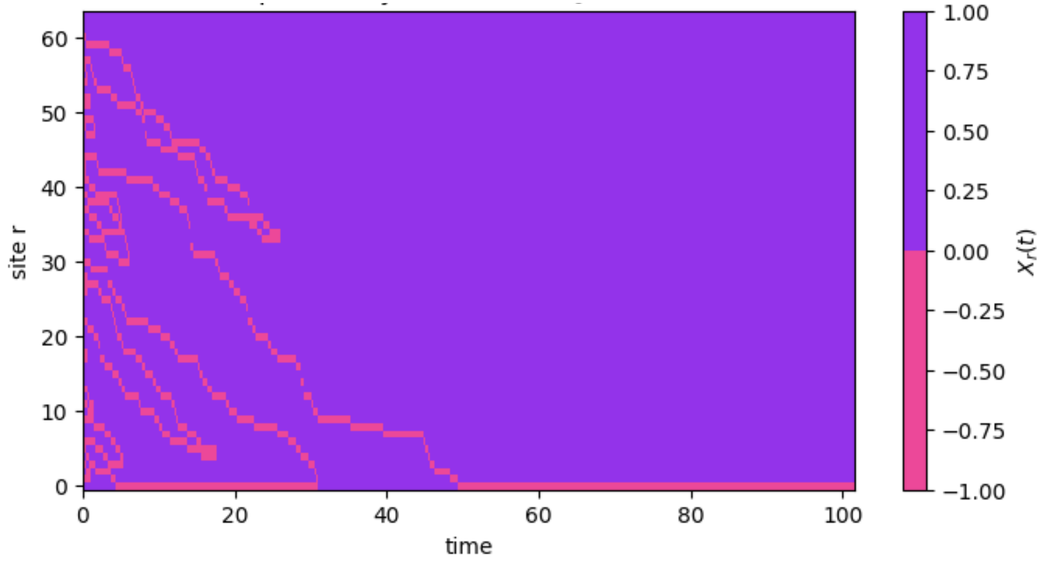


Parent Lindbladian doesn't need to be gapless! But has to be slow.

$\gamma = 0$: gapped, but still $\mathcal{O}(L)$ mixing time

$\rho_- = |-+\cdots+\rangle\langle-+\cdots+|$

(only possible by breaking translation sym)



Phases with strong Z2 symmetry: STSSB

$\rho \propto (1+U)$

Strong-to-weak SSB

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Strongly symmetric

$\rho = |\mathrm{GHZ}+\rangle\langle\mathrm{GHZ}+|$

Strong-to-trivial SSB

STSSB : slow, symmetric parent Lindbladian

$$\mathscr{L}_r^{+}[\rho] = \frac{(1+X_r)}{2}\rho\frac{(1+X_r)}{2} + Z_{r-1}Z_r\frac{(1-X_r)}{2}\rho\frac{(1-X_r)}{2}Z_{r-1}Z_r - \rho$$

↓ KW duality on jump ops

$$\mathscr{L}_r^{\mathrm{STSSB}}[\rho] = \frac{(1+Z_rZ_{r+1})}{2}\rho\frac{(1+Z_rZ_{r+1})}{2} + X_r\frac{(1-Z_rZ_{r+1})}{2}\rho\frac{(1-Z_rZ_{r+1})}{2}X_r - \rho$$

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$$\rho_{-} = |\mathrm{GHZ}-\rangle\langle\mathrm{GHZ}-|$$

SWSSB diagnostic: fidelity correlator

Two-point fidelity correlator: $F_{ij}(\rho, Z_i Z_j) = \mathrm{Tr}(Z_i Z_j \sqrt{\rho} Z_i Z_j \sqrt{\rho})$ Weinstein 2025; Liu/Chen/Zhang/Zhou/Zhang 2025

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One-point correlator nonzero by strong symmetry:

$$F_i(\rho, Z_i) = \mathrm{Tr}(Z_i\sqrt{\rho}Z_i\sqrt{\rho})$$

$$= \mathrm{Tr}(Z_iU\sqrt{\rho}Z_i\sqrt{\rho})$$

$$= -\mathrm{Tr}(Z_i\sqrt{\rho}Z_i\sqrt{\rho}) \rightarrow F_i(\rho, Z_i) = 0$$

$F_{ij}(\rho, Z_iZ_j) > 0 \rightarrow$ strong symmetry SSB'd but not necessarily weak symmetry

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$\rho \propto (1+U)$	$\rho = \vert+\rangle\langle+\vert$	$\rho = \vert\mathrm{GHZ}+\rangle\langle\mathrm{GHZ}+\vert$
Strong-to-weak SSB	Strongly symmetric	Strong-to-trivial SSB
$F_{ij}(\rho, Z_iZ_j) = 1$	$F_{ij}(\rho, Z_iZ_j) = 0$	$F_{ij}(\rho, Z_iZ_j) = 1$

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Stability Theorem: Lessa/Ma/Zhang/Bi/Cheng/Wang 2025

If ρ has $F_{ij}(\rho, O_i O_j^\dagger) > 0$ for some charged O_i, O_j then $F_{ij}(\mathscr{E}[\rho], \tilde{O}_i \tilde{O}_j^\dagger) > 0$

for $\mathscr{E}$ any symmetric finite depth quantum channel.

SWSSB diagnostic: fidelity correlator

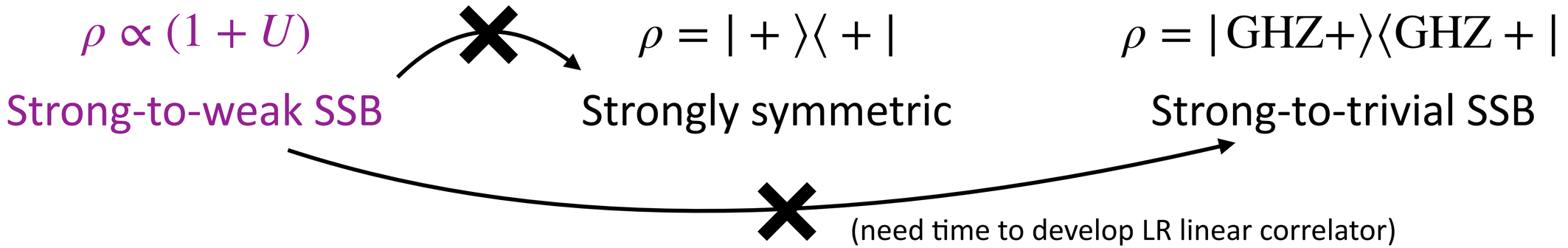
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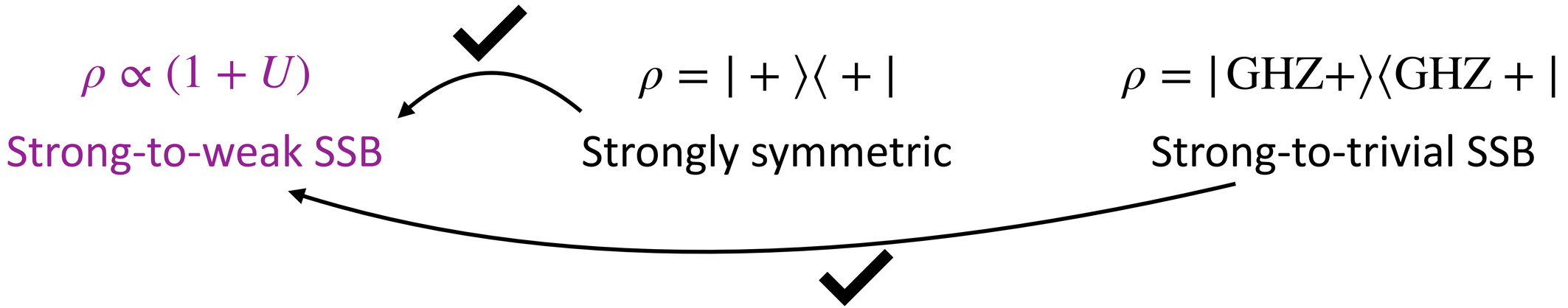
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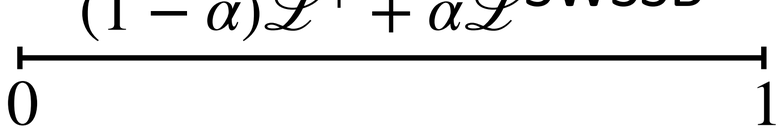
for $\mathscr{E}$ any symmetric finite depth quantum channel.



Phase transitions?

$\rho = |+\rangle\langle+|$

Strongly symmetric



$\rho \propto 1 + U$

SWSSB

Phase transitions?

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Strongly symmetric

$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\text{SWSSB}}$

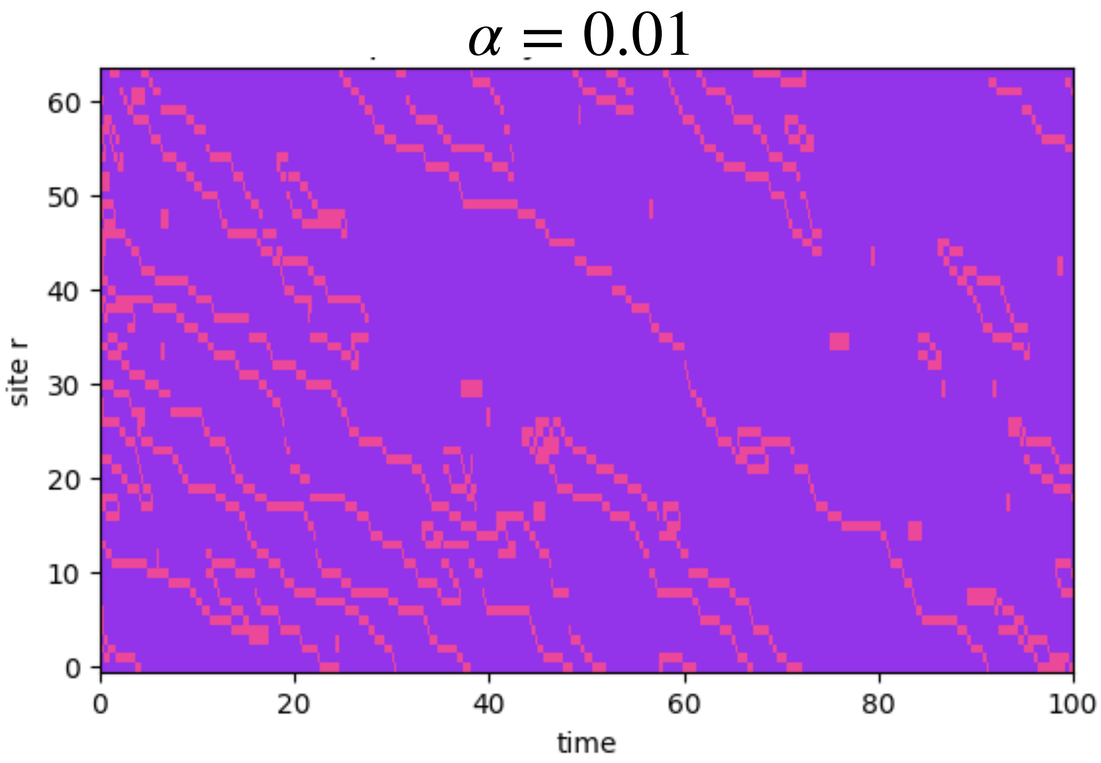
0 — 1

$\rho \propto 1 + U$

SWSSB

$\mathscr{L}^{+}$ is unstable to $\mathscr{L}^{\text{SWSSB}}$!

- defects spawn spontaneously
- defects cleaned diffusively



Generic in any dimension

$1+1d$: STSSB phase unstable, strongly symmetric phase unstable

$\geq 2+1d$: STSSB phase stable, strongly symmetric phase unstable

ferromagnetic phase

$$\rho \propto \frac{1+\prod_r X_r}{2} e^{-\beta H}$$

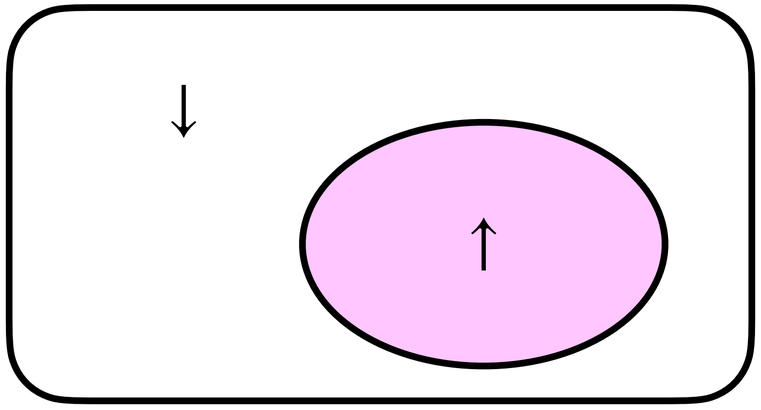
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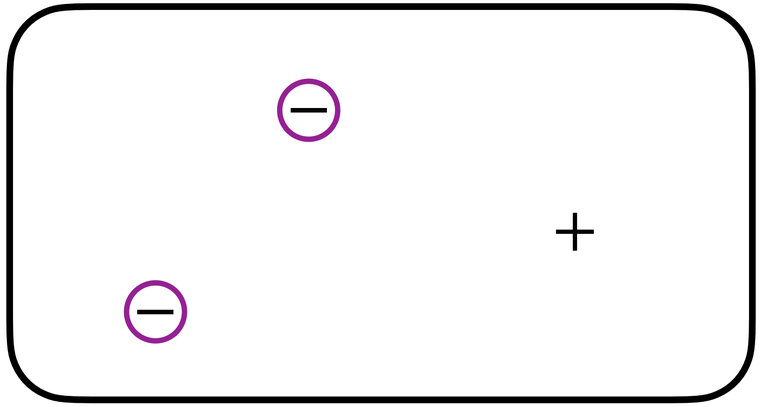
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STSSB: efficient domain wall erosion



Symmetric: point-wise annihilation :(

Generic in any dimension

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ferromagnetic phase

$$\rho \propto \frac{1+\prod_r X_r}{2} e^{-\beta H}$$

Can prove: any projected finite temp Gibbs state has $F_{ij}(\rho, Z_i Z_j) \geq e^{-\beta \|V\|/2} > 0$

$$V = Z_i Z_j H Z_i Z_j - H$$

Generic in any dimension

Intuition from classical Gibbs state: $\rho = \frac{1}{Z} e^{-\beta H}, H = \sum_r a_r X_r + \sum_{r,r'} b_{r,r'} X_r X_{r'} + \cdots$

Generic in any dimension

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$$F_{ij}(\rho, Z_i Z_j) = \mathrm{Tr}\left(Z_i Z_j \sqrt{\rho} Z_i Z_j \sqrt{\rho} \right) = \frac{1}{Z} \mathrm{Tr}\left(e^{-\beta H/2} e^{-\beta Z_i Z_j H Z_i Z_j/2} \right) \qquad \sqrt{\rho} = \frac{1}{\sqrt{Z}} e^{-\beta H/2}$$

Generic in any dimension

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$$= \frac{1}{Z} \mathrm{Tr}\left(e^{-\beta H} e^{-\beta (Z_i Z_j H Z_i Z_j - H)/2} \right)$$

$$\sqrt{\rho} = \frac{1}{\sqrt{Z}} e^{-\beta H/2}$$

Generic in any dimension

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$$= \frac{1}{Z} \mathrm{Tr}\left(e^{-\beta H} e^{-\beta (Z_i Z_j H Z_i Z_j - H)/2} \right)$$

$$= \langle e^{-\beta V/2} \rangle_\rho > 0 \text{ (for finite } \beta, \|V\|) \qquad V = Z_i Z_j H Z_i Z_j - H$$

Remains positive with symmetry projector

Generic in any dimension

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Can prove: any projected finite temp Gibbs state has $F_{ij}(\rho, Z_i Z_j) \geq e^{-\beta \|V\|/2} > 0$

$$V = Z_i Z_j H Z_i Z_j - H$$

Need to go beyond Gibbs states/Gibbs samplers to get phase transitions!

Weakening the SWSSB phase

Design dynamics such that

- $|+\rangle\langle+| \to \rho_0 = |+\rangle\langle+|$
- any other even parity state $\to \rho_+ \propto \frac{1+\prod_r X_r}{2} - |+\rangle\langle+|$

$\Big\}\, U = +1$

- any odd parity state $\to \rho_- \propto \frac{1-\prod_r X_r}{2}$

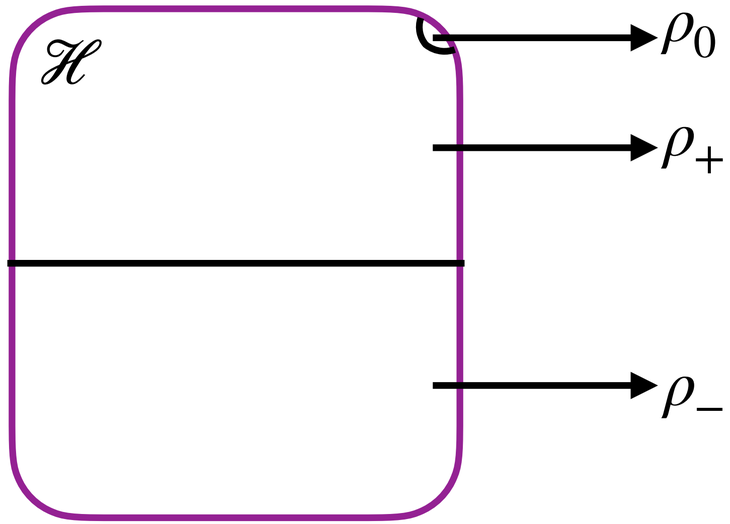
$\Big\}\, U = -1$

$$\text{Steady state space} = \text{span}(\rho_0, \rho_-, \rho_+)$$

"SWSSB + symmetric" in even sector, SWSSB in odd sector

Weakening the SWSSB phase

Design dynamics such that



SWSSB at late times

with prob $1 - 2^{-L}$

Steady state space $=$ span(ρ_0, ρ_-, ρ_+)

"SWSSB + symmetric" in even sector, SWSSB in odd sector

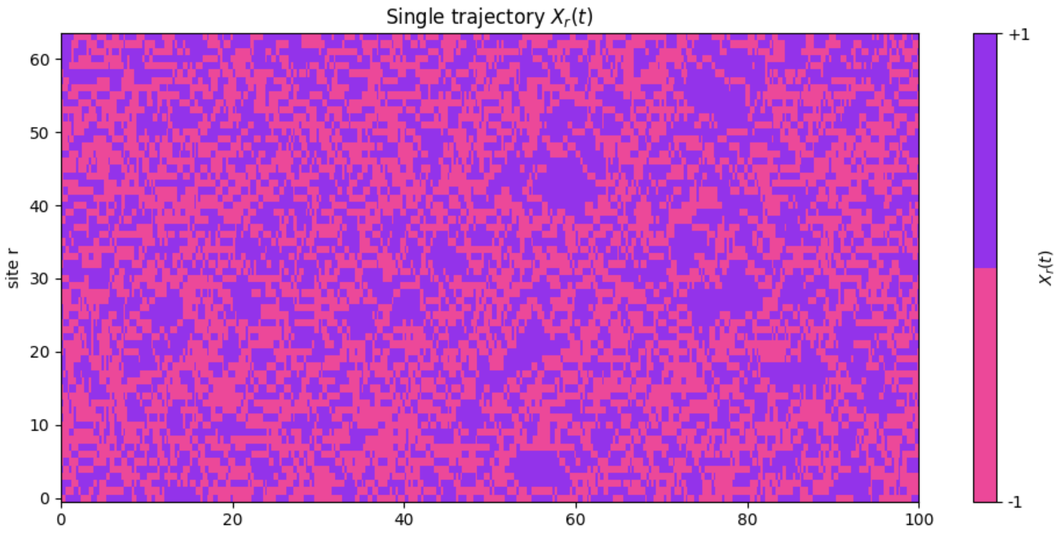
Weakening the SWSSB phase

Three site update rule (after X dephasing) :

$$(+++) \rightarrow (+++)$$

scramble $(-++), (+-+), (++-), (---)$

scramble $(--+), (-+-), (+--)$



Weakening the SWSSB phase

Three site update rule (after X dephasing) :

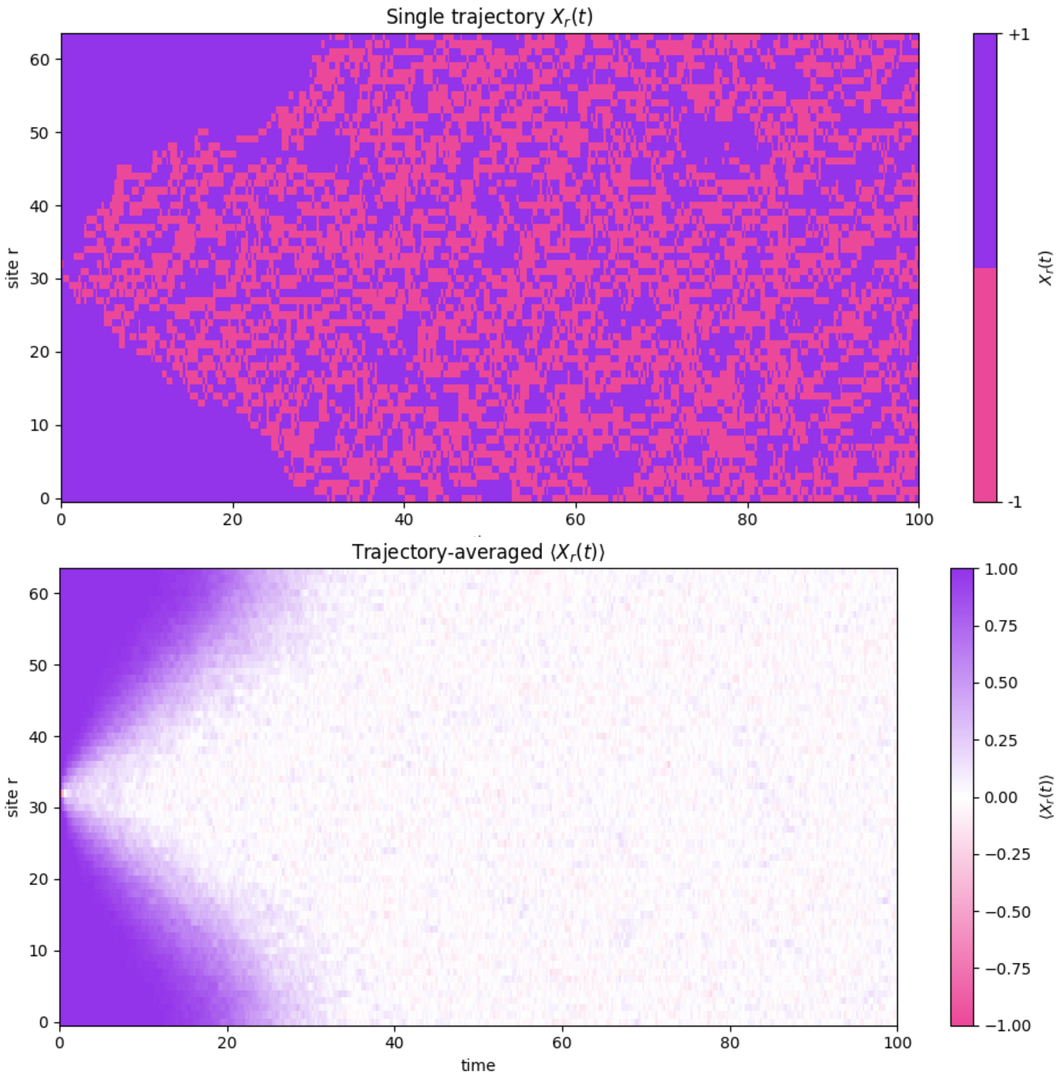
$$(+++) \to (+++)$$

scramble $(-++), (+-+), (++-), (---)$

scramble $(--+), (-+-), (+--)$

No more spawning defects spontaneously! ☺

$\mathcal{O}(L)$ mixing time

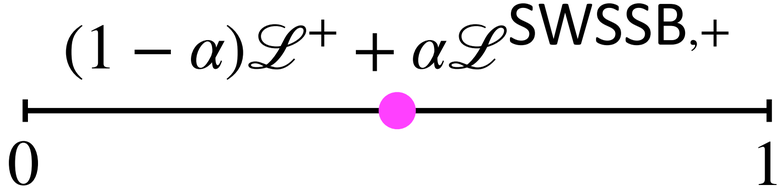


A genuine phase transition

$$\rho_0 = |+\rangle\langle+|$$

$$\rho_W \propto \sum_r Z_r |+\rangle\langle+| Z_r$$

Strongly symmetric



$$\rho_+ \propto \frac{1+U}{2} - \rho_0$$

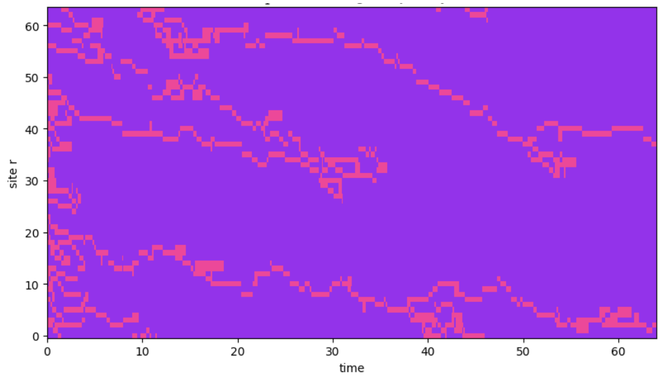
$$\rho_0 = |+\rangle\langle+|$$

$$\rho_- \propto \frac{1-U}{2}$$

SWSSB + symmetric

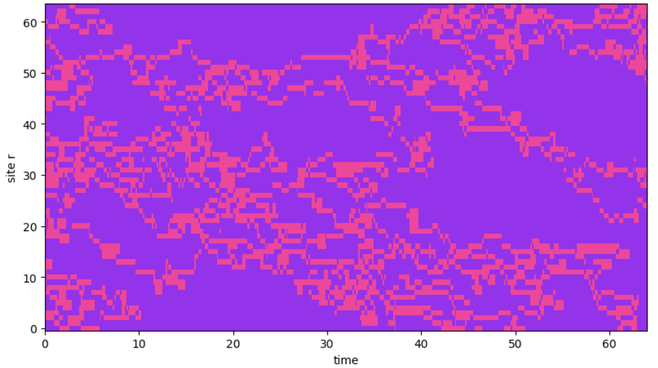
A genuine phase transition

def late time defect density: $n_- = \frac{1}{L}\sum_r \frac{1-\langle X_r \rangle}{2}$



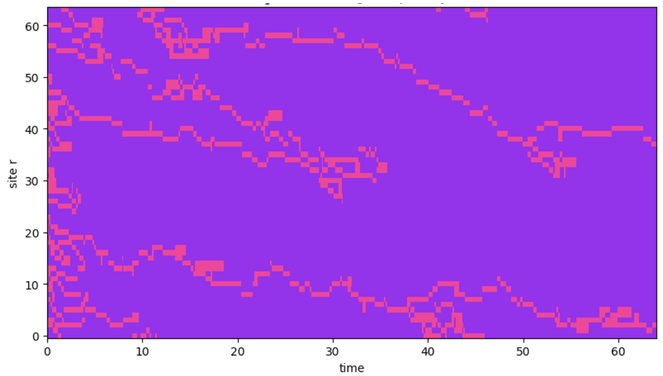
$$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\mathrm{SWSSB},+}$$

$$0 \qquad \alpha_c \sim 0.44 \qquad 1$$



A genuine phase transition

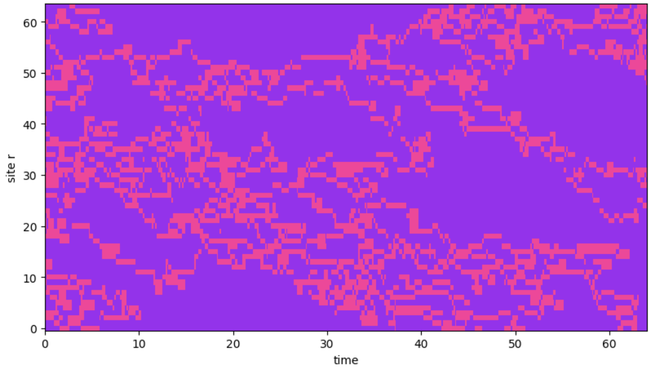
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$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\text{SWSSB},+}$

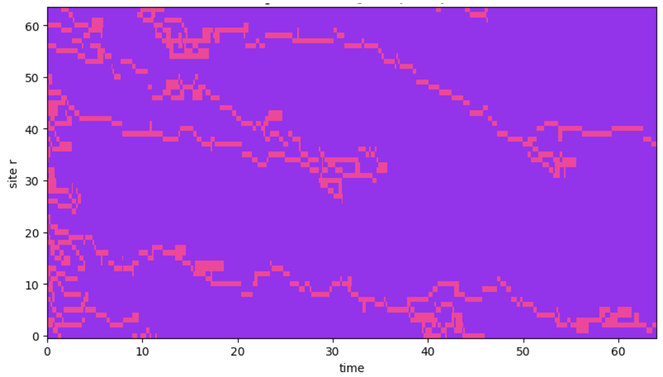
0 — 1

$\lim_{L\to\infty} n_-(L) = 0$ $\lim_{L\to\infty} n_-(L) > 0$



A genuine phase transition

def late time defect density: $n_- = \frac{1}{L}\sum_r \frac{1-\langle X_r \rangle}{2}$



$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\mathrm{SWSSB},+}$

0

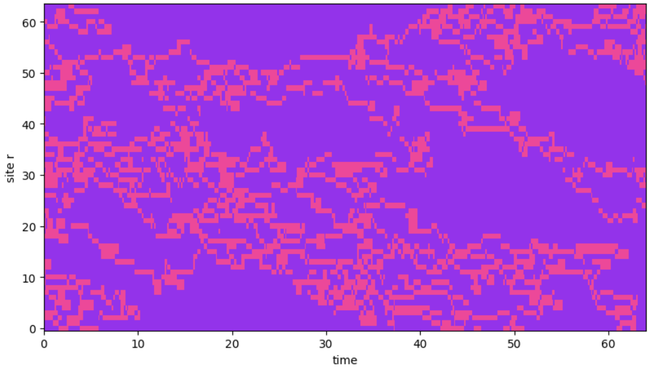
$\alpha_c \sim 0.44$

1

gapless

metastable SWSSB

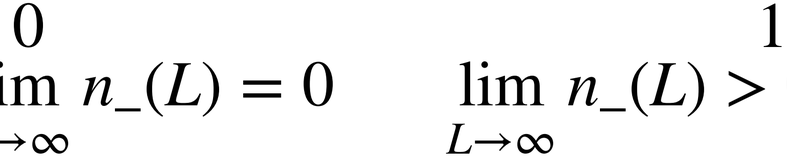
SWSSB steady states



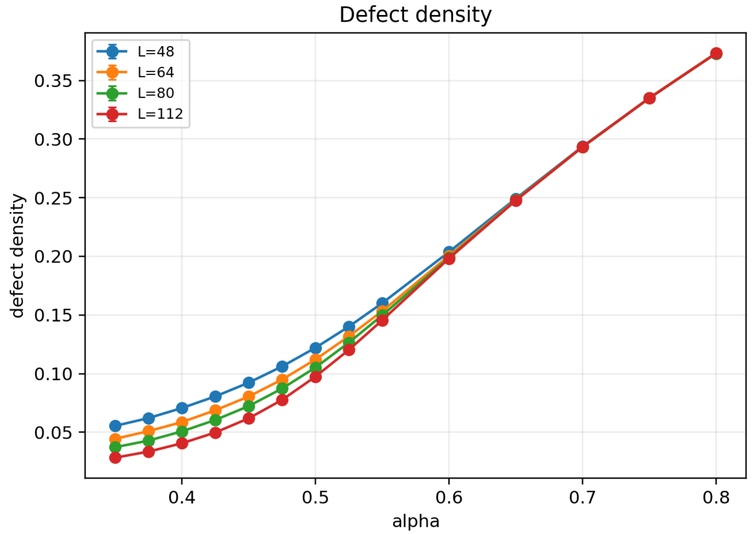
A genuine phase transition

def late time defect density: $n_ = \frac{1}{L}\sum_r \frac{1-\langle X_r\rangle}{2}$

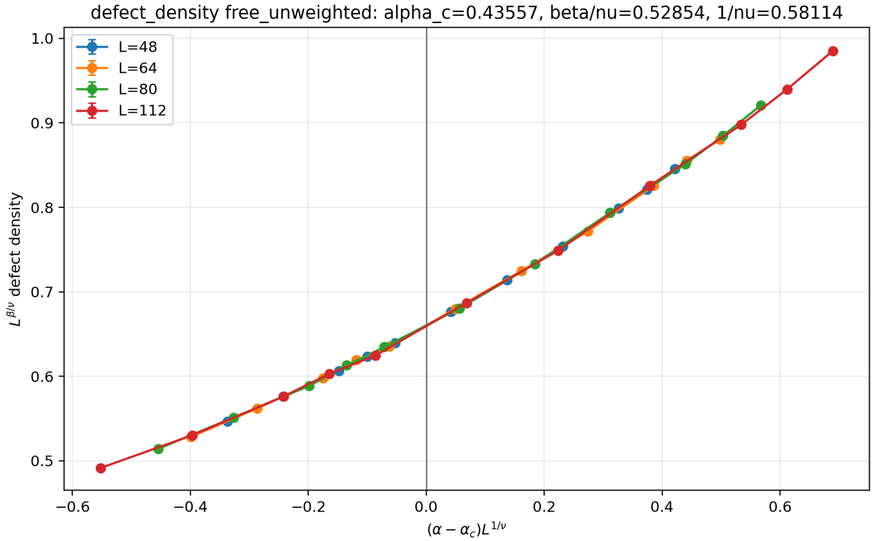
$$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\text{SWSSB},+}$$



$$\lim_{L\to\infty} n_(L) = 0 \qquad \lim_{L\to\infty} n_(L) > 0$$



Universality class of the transition

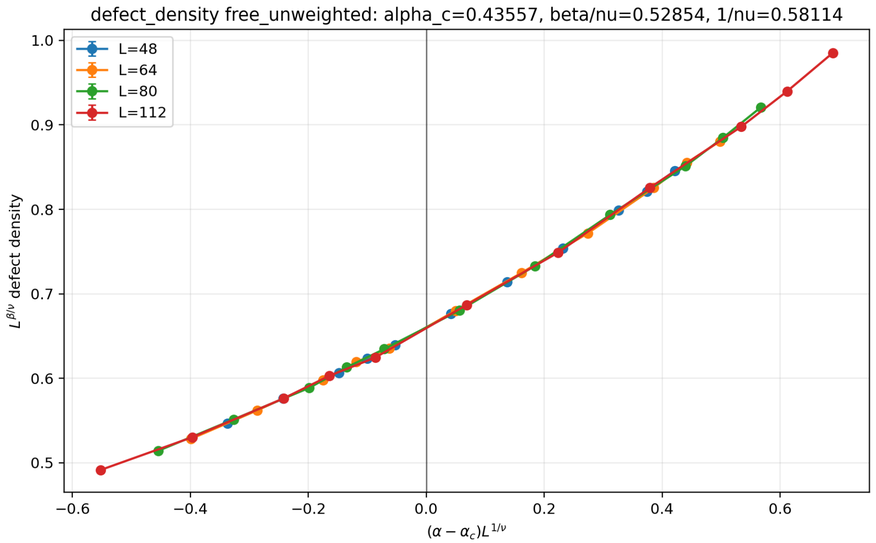


Close to parity-conserving branching annihilating random walks:

$$\beta \sim 0.93 \quad \beta/\nu_\perp \sim 1/2 \quad 1/\nu_\perp \sim 0.54$$ i.e. Jensen PRE 1994

absorbing $\sim$ symmetric, active $\sim$ SWSSB

Universality class of the transition



Close to parity-conserving branching annihilating random walks:

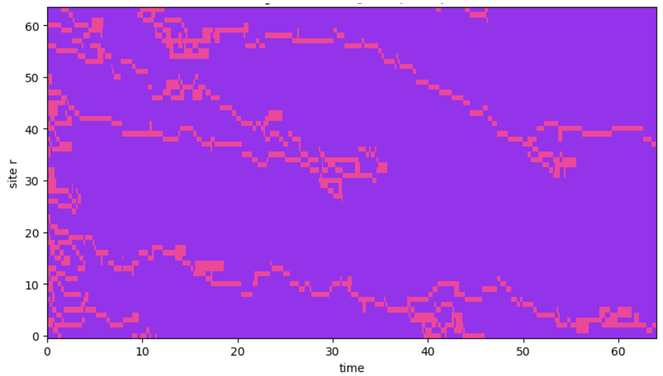
$$\beta \sim 0.93 \quad \beta/\nu_\perp \sim 1/2 \quad 1/\nu_\perp \sim 0.54$$ i.e. Jensen PRE 1994

absorbing $\sim$ symmetric, active $\sim$ SWSSB

Generic for $2A \to \varnothing$ annihilation, $A \to 3A$ branching

Diagnosing the SWSSB/“active” phase

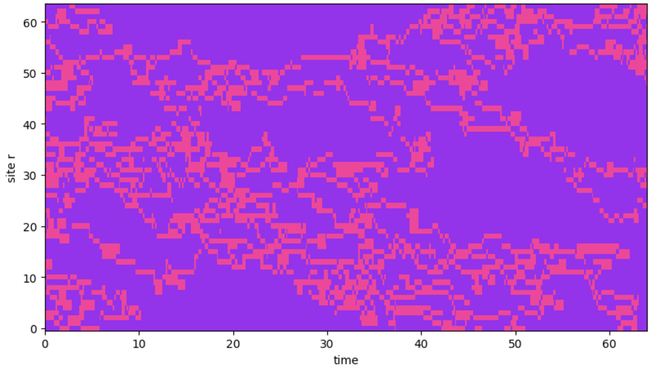
def late time defect density: $n_- = \frac{1}{L}\sum_r \frac{1-\langle X_r \rangle}{2}$



$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\text{SWSSB},+}$

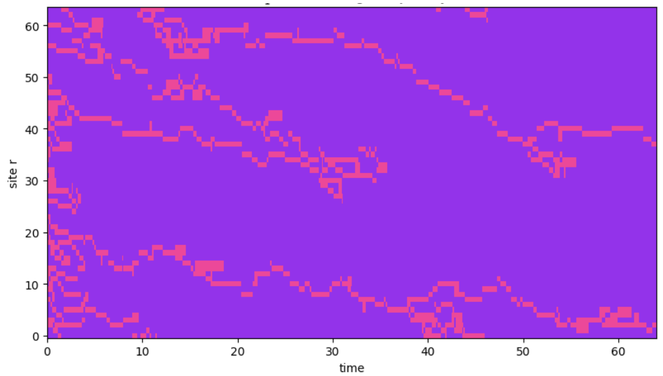
0 — 1

$\lim_{L\to\infty} n_-(L) = 0$ $\quad$ $\lim_{L\to\infty} n_-(L) > 0$



Diagnosing the SWSSB/“active” phase

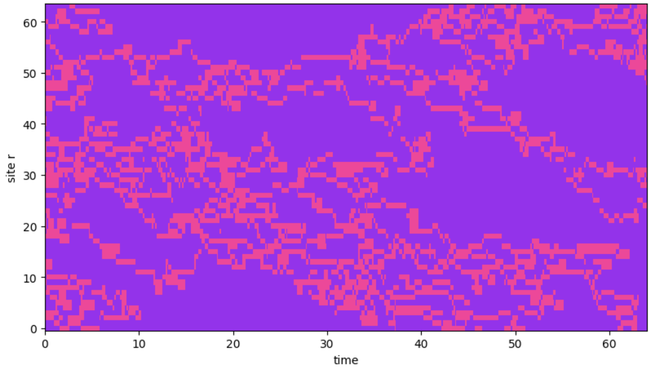
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$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\text{SWSSB},+}$

0 — 1

$\lim_{L\to\infty} n_-(L) = 0$ $\quad$ $\lim_{L\to\infty} n_-(L) > 0$



In principle can have product state with $n_- > 0$!

Need both $n_- > 0$ and fluctuations!

What about the fidelity correletor $\mathrm{Tr}(Z_i Z_j \sqrt{\rho} Z_i Z_j \sqrt{\rho})$?

Diagnosing the SWSSB/“active” phase

For ρ diagonal in the X basis:

$$F_{ij}(\rho, Z_i Z_j) = \sum_{\{X_r\}} \sqrt{p(\{X_r\}) p(\{X_r^{ij}\})}$$

Bhattacharyya distance of classical probabilities

flip signs of X_i, X_j

Diagnosing the SWSSB/“active” phase

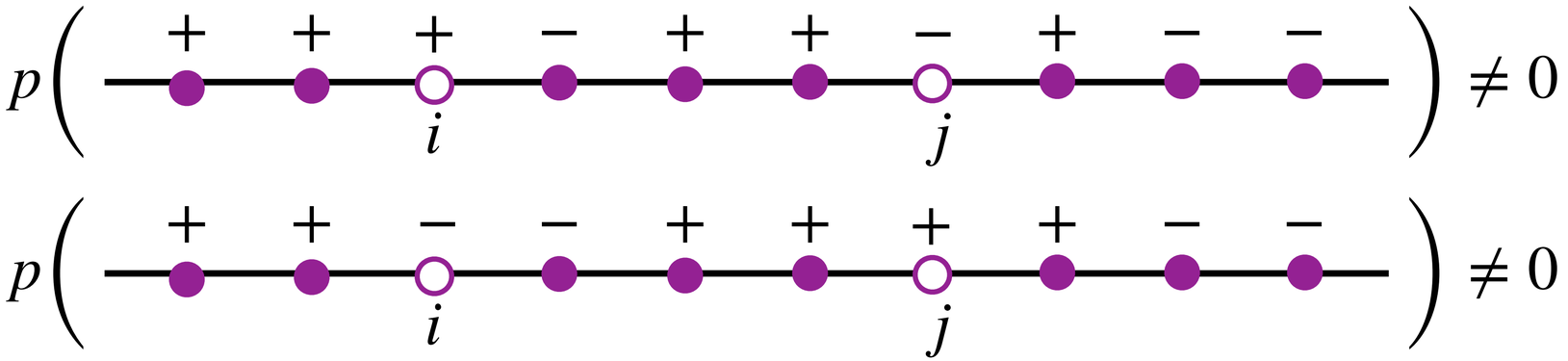
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Fixing $\{X_{r\neq i,j}\}$, can X_i, X_j fluctuate?



Diagnosing the SWSSB/“active” phase

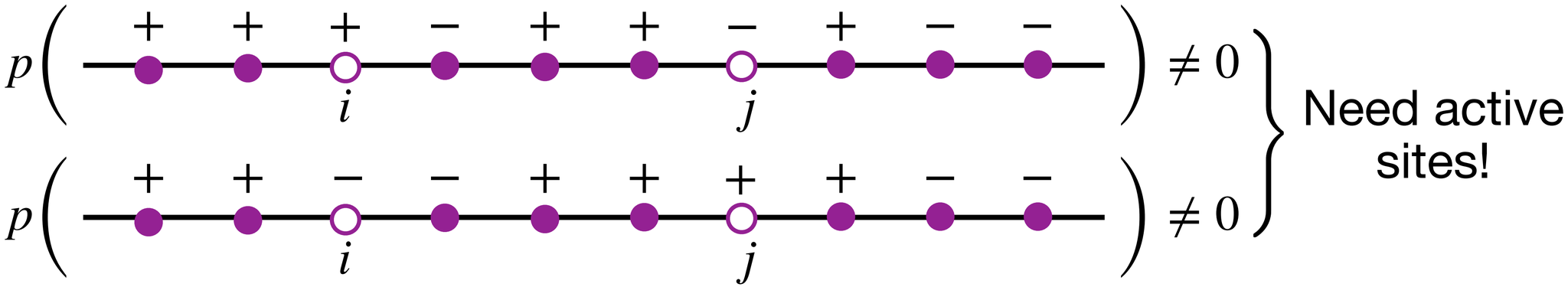
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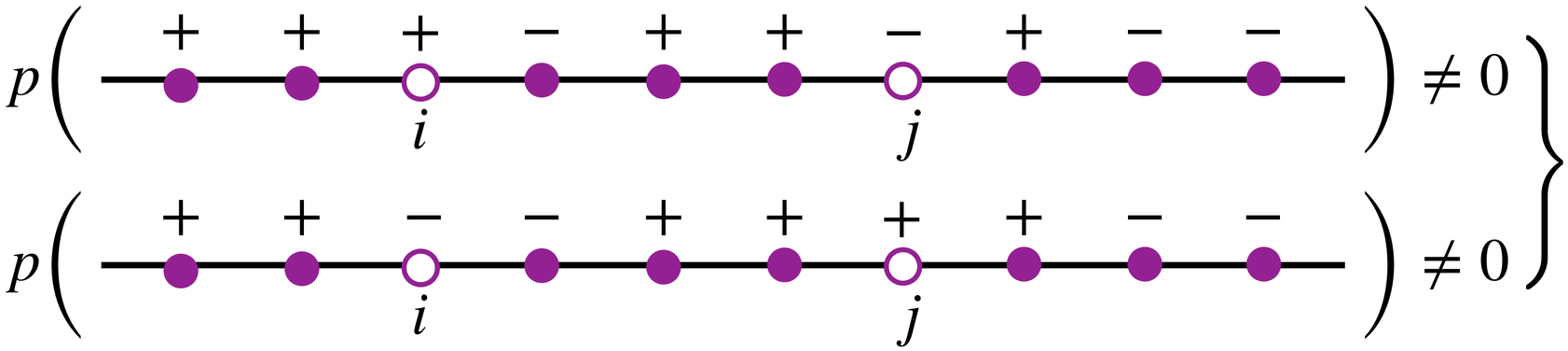
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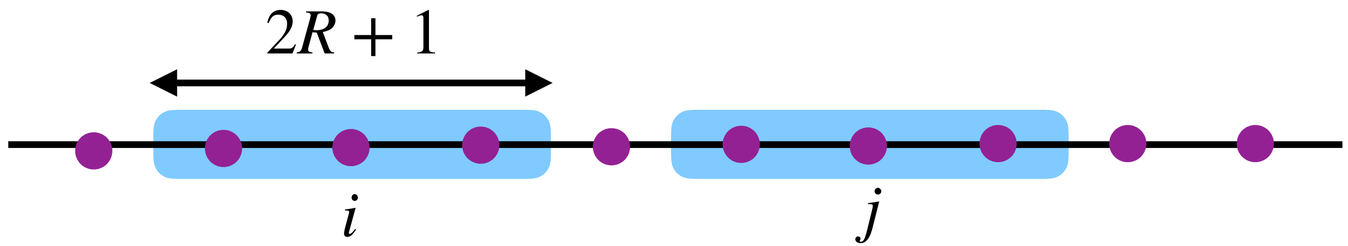
Fixing $\{X_{r\neq i,j}\}$, can X_i, X_j fluctuate?



However, need $\mathcal{O}(2^V)$ samples to measure :(

Marginal radius-R fidelity

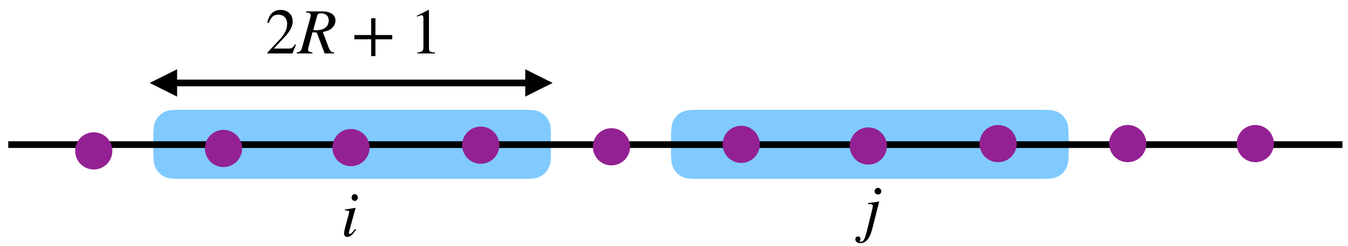
$$F_{ij}^{R}(\rho, Z_i Z_j) = \mathrm{Tr}\left(Z_i Z_j \sqrt{\rho_{R,ij}} Z_i Z_j \sqrt{\rho_{R,ij}}\right) \qquad \rho_{R,ij} = \mathrm{Tr}_{\overline{R(i) \cup R(j)}}(\rho)$$



Only requires tomography on a size $2(2R+1)$ region

Marginal radius-R fidelity

$$F_{ij}^{R}(\rho, Z_i Z_j) = \mathrm{Tr}\left(Z_i Z_j \sqrt{\rho_{R,ij}} Z_i Z_j \sqrt{\rho_{R,ij}} \right) \qquad \rho_{R,ij} = \mathrm{Tr}_{\overline{R(i) \cup R(j)}}(\rho)$$



Only requires tomography on a size $2(2R+1)$ region

If mutual information $S(A) + S(B) - S(AB)$ decays exponentially,

$$F_{ij}^{R}(\rho, Z_i Z_j) \sim F_{i}^{R}(\rho, Z_i) F_{j}^{R}(\rho, Z_j) \sim (F_{i}^{R}(\rho, Z_i))^2$$

Only requires tomography on a size $(2R+1)$ region

Properties of marginal fidelity

Monotonicity:

$F_{ij}^{R}(\rho, Z_iZ_j) \leq F_{ij}^{R'}(\rho, Z_iZ_j)$ if $R > R' : F_{ij}^{0} \geq F_{ij}^{1} \geq F_{ij}^{2} \geq \cdots F_{ij}$

In particular if there is SWSSB, $F_{ij}^{R}(\rho, Z_iZ_j) > 0$, and if $F_{ij}^{R}(\rho, Z_iZ_j) = 0$ there is no SWSSB.

Stability

Inherits stability theorems from true fidelity as long as $R > \mathcal{O}$(depth of channel)

Complexity

Strong symmetry $+ F_{i}^{R}(\rho, Z_i) > 0$ for large R implies long range CMI

$+$ Can proves fast convergence in R for broad classes of states

Summary

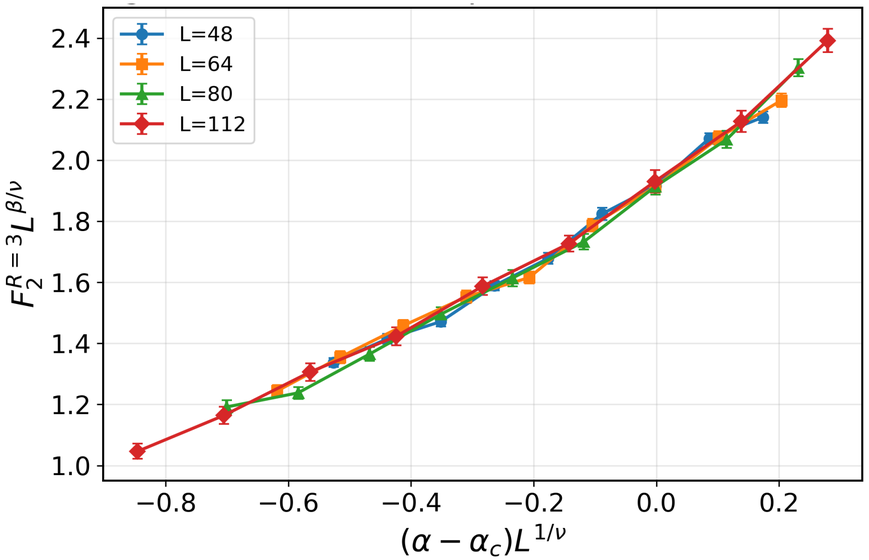
Marginal fidelity correlator is usually a good proxy for true fidelity correlator.

It satisfies several nice information theoretic properties.

Can it be applied generally to detect transitions into an active phase?

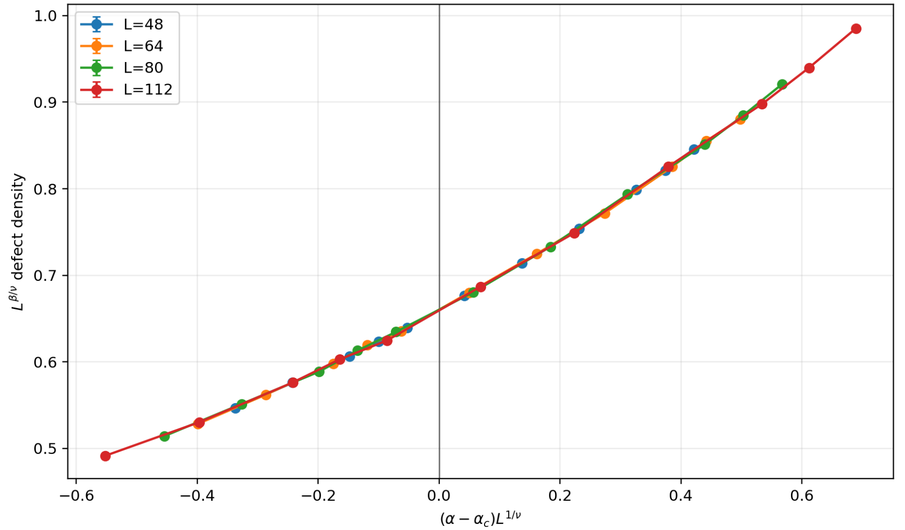
Marginal 2-point fidelity at the transition

$F_{ij}^2(\rho, Z_i Z_j)$



$\alpha_c = 0.46, \beta/\nu_\perp = 0.52, 1/\nu_\perp = 0.56$

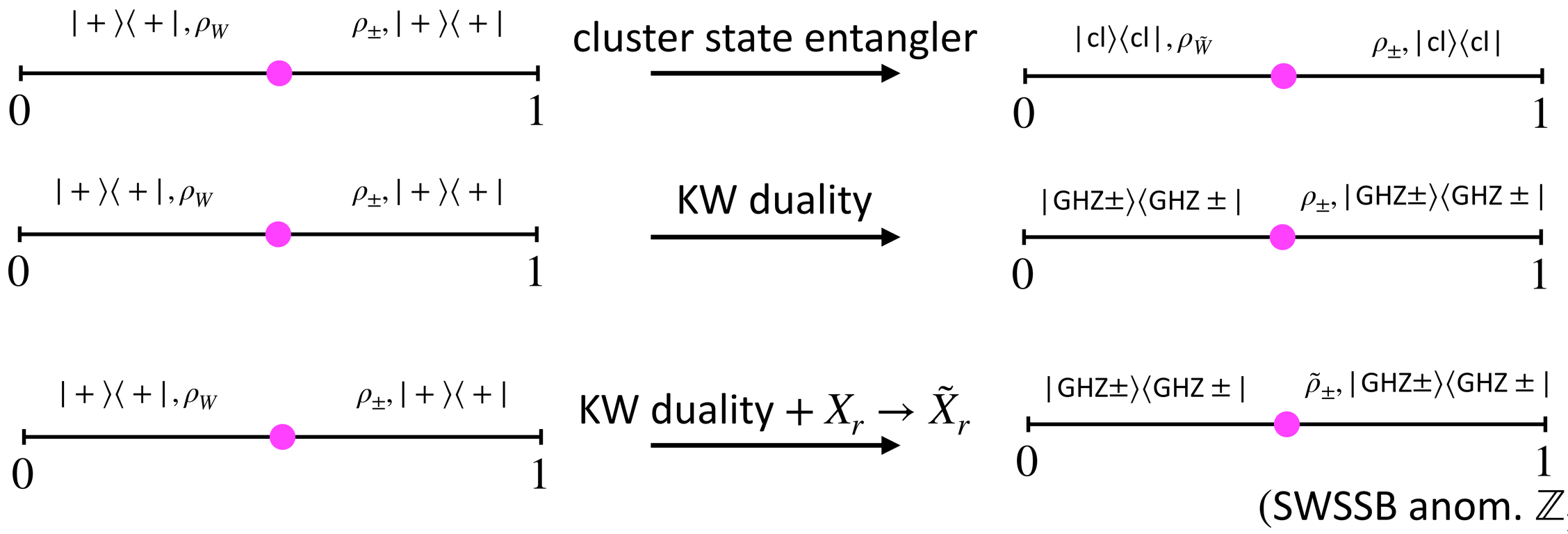
n_- (defect density)



$\alpha_c = 0.44, \beta/\nu_\perp = 0.53, 1/\nu_\perp = 0.58$

Re: literature on PC-BARW: $\beta/\nu_\perp \sim 1/2$ $1/\nu_\perp \sim 0.54$

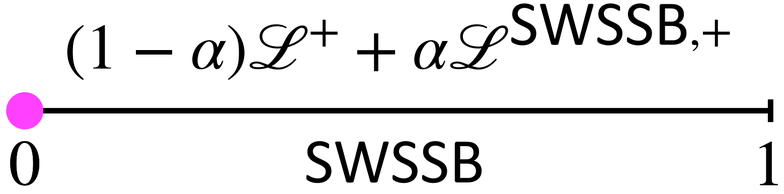
Other SWSSB transitions



2+1d phase transitions

Problem: no symmetric phase with $2A \to \varnothing$ annihilation, $A \to 3A$ branching in $\geq$ 2+1d!

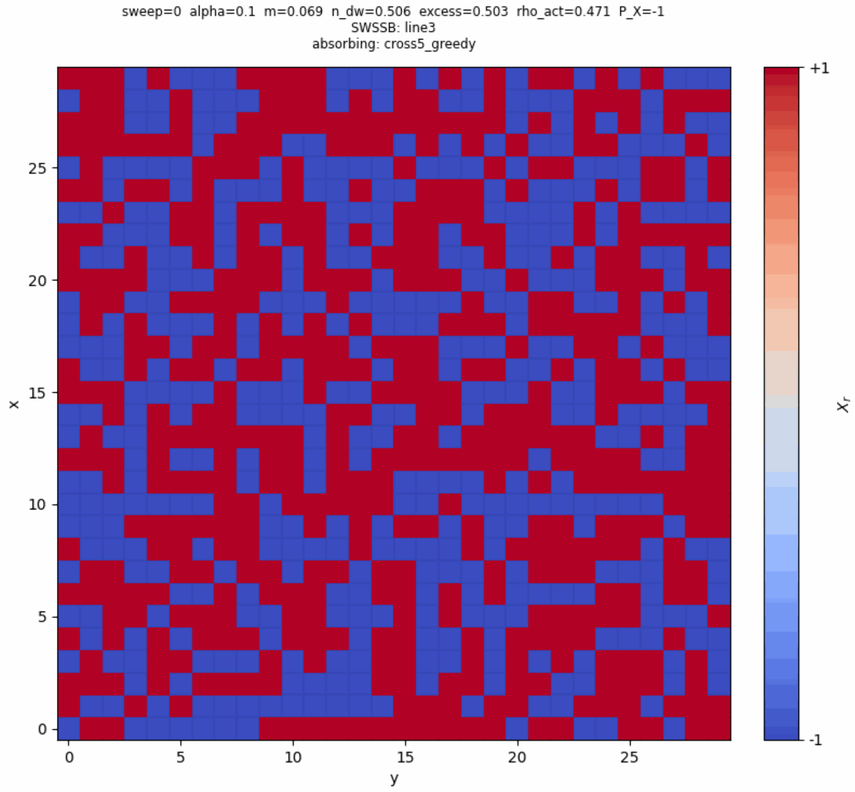
Cardy and Täuber 1998



2+1d phase transitions

Problem: no symmetric phase with $2A \to \varnothing$ annihilation, $A \to 3A$ branching in $\geq$ 2+1d!

Cardy and Täuber 1998



Aggressive 5-site annihilation rule, with 10% chance of 1->3 branching

Solution: add another steady state

Design dynamics such that

- $|+\rangle\langle+| \rightarrow \rho_0 = |+\rangle\langle+|$
- $|-\rangle\langle-| \rightarrow \rho_0^- = |-\rangle\langle-|$
- any other even parity state $\rightarrow \rho_+ \propto \frac{1+\prod_r X_r}{2} - \rho_0 - \rho_0^-$

$\Big\}\ U = +1$ (even number of sites)

- any odd parity state $\rightarrow \rho_- \propto \frac{1-\prod_r X_r}{2}$

$\Big\}\ U = -1$

$$\text{Steady state space} = \text{span}(\rho_0, \rho_0^-, \rho_-, \rho_+)$$

Solution: add another steady state

Design dynamics such that

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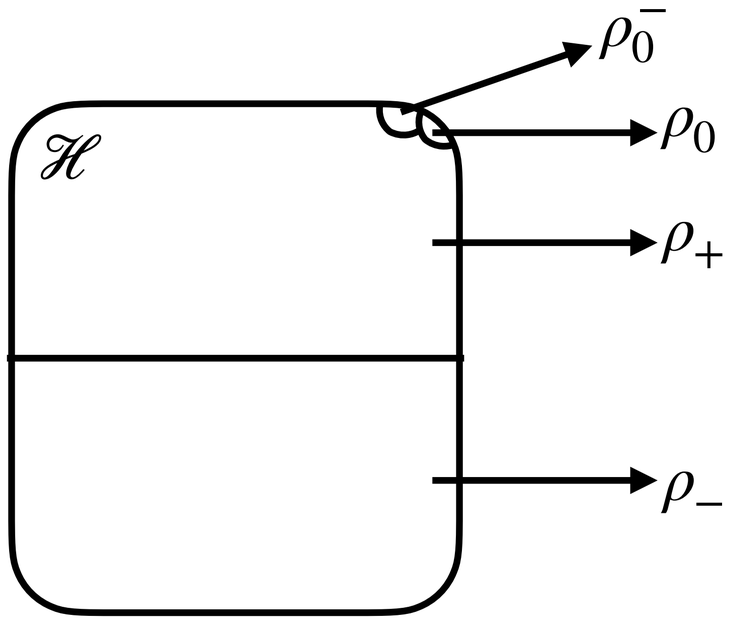
$\Big\}$ $U = -1$

$$\text{Steady state space} = \text{span}(\rho_0, \rho_0^-, \rho_-, \rho_+)$$

Main idea: use domain wall erosion rather than pairwise particle annihilation

Solution: add another steady state

Design dynamics such that



SWSSB at late times

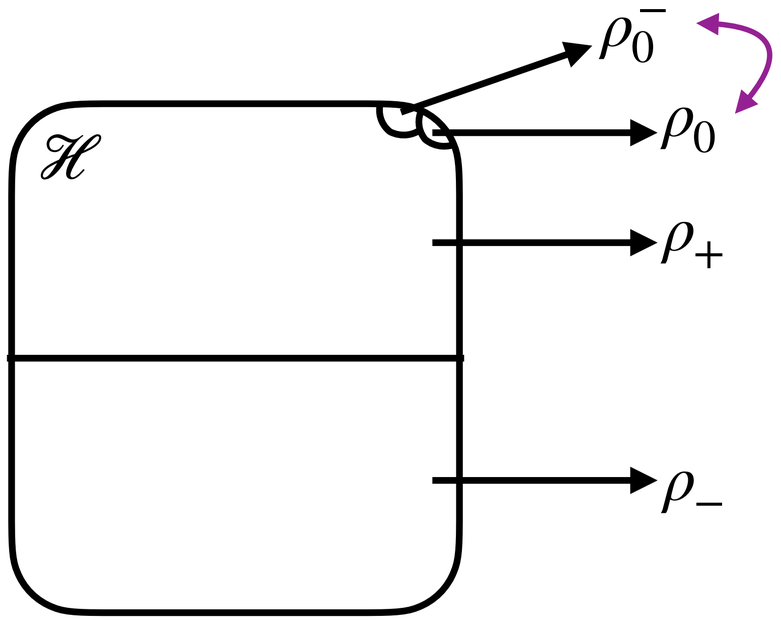
with prob $1 - 2 \cdot 2^{-L}$

$$\text{Steady state space} = \text{span}(\rho_0, \rho_0^-, \rho_-, \rho_+)$$

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SWSSB at late times

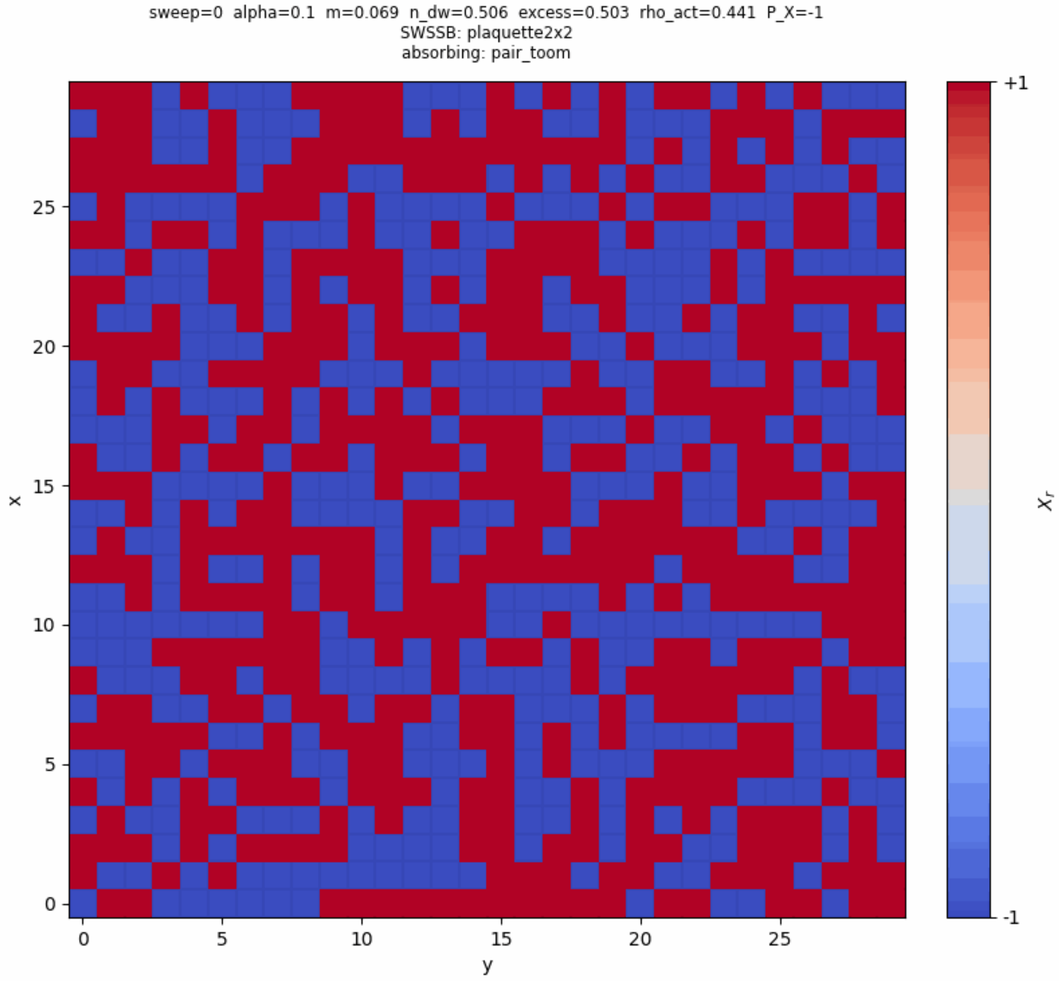
with prob $1 - 2 \cdot 2^{-L}$

New ingredient: weak $\mathbb{Z}_2$ symmetry $\prod_r Z_r$

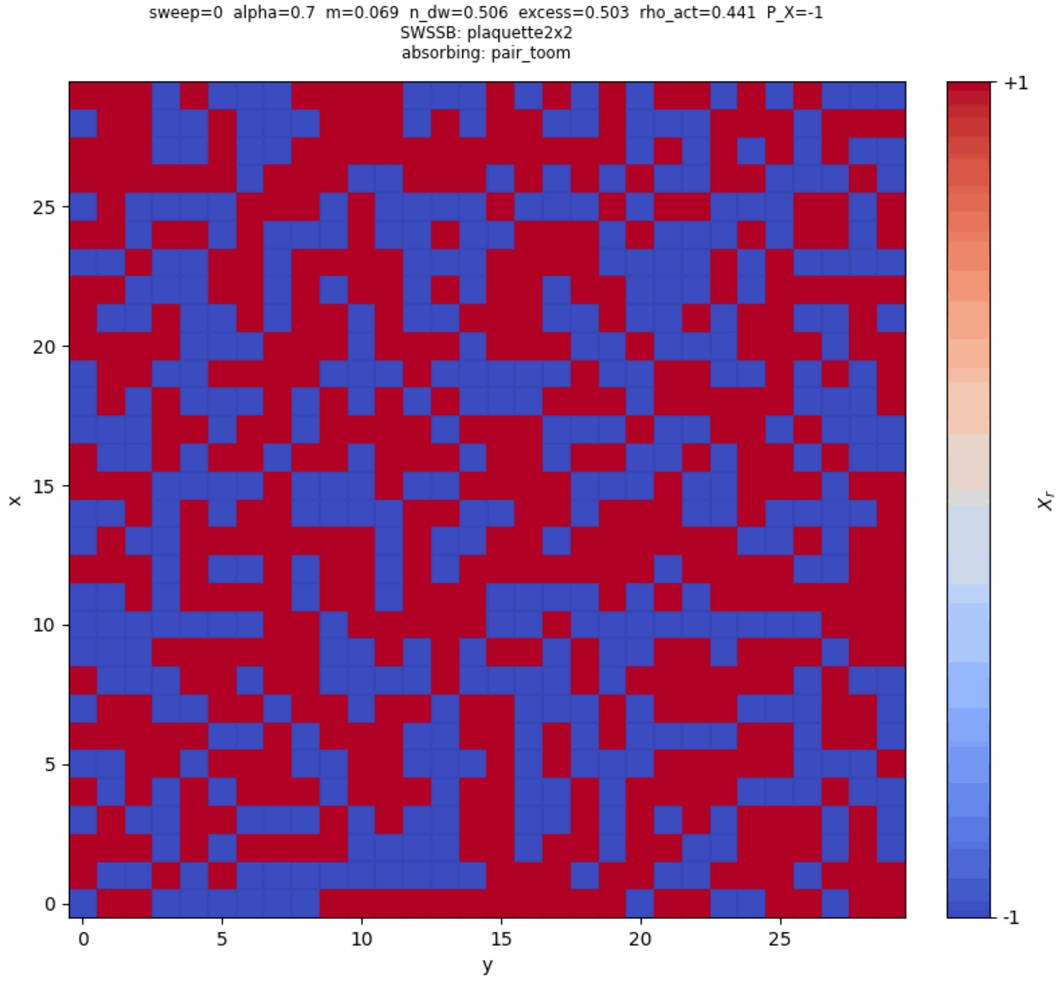
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Main idea: use domain wall erosion rather than pairwise particle annihilation

Symmetric phase

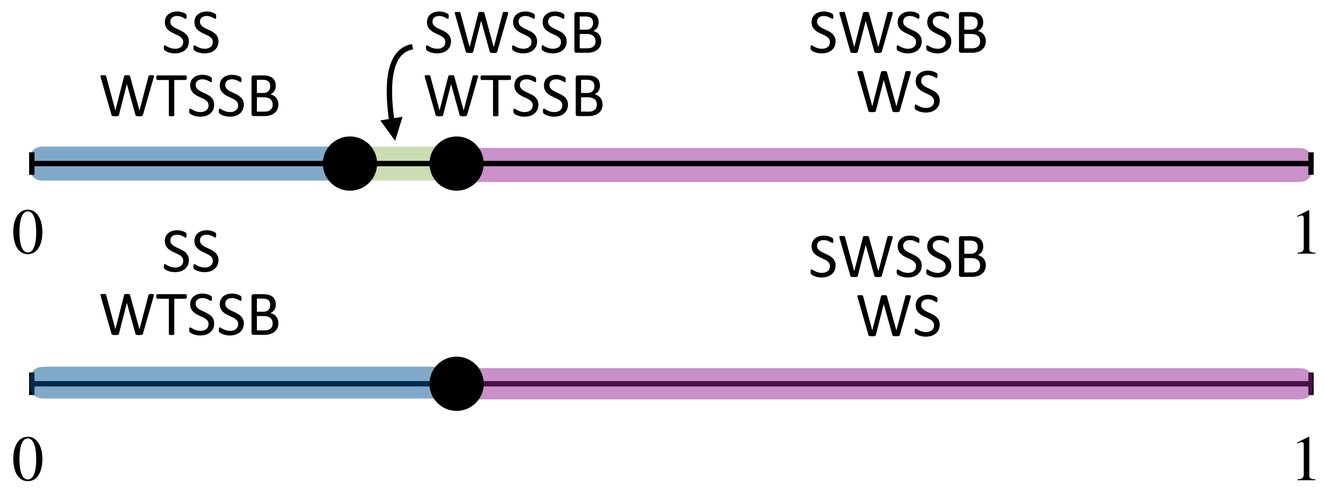


SWSSB phase


sweep=0 alpha=0.7 m=0.069 n_dw=0.506 excess=0.503 rho_act=0.441 P_X=-1
SWSSB: plaquette2x2
absorbing: pair_toom
x
y
0
5
10
15
20
25
+1
-1
X_r

Weak Z2 symmetry: multiple transitions?

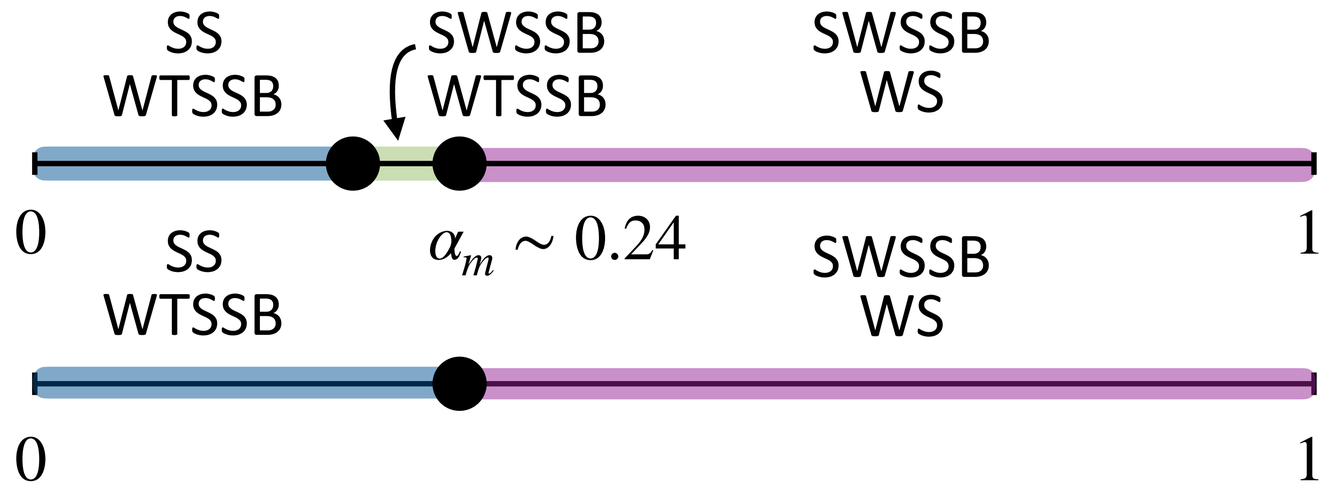
ρ_{WTSSB}
$\propto |+\rangle\langle+| + |-\rangle\langle-|$
(even number of sites)



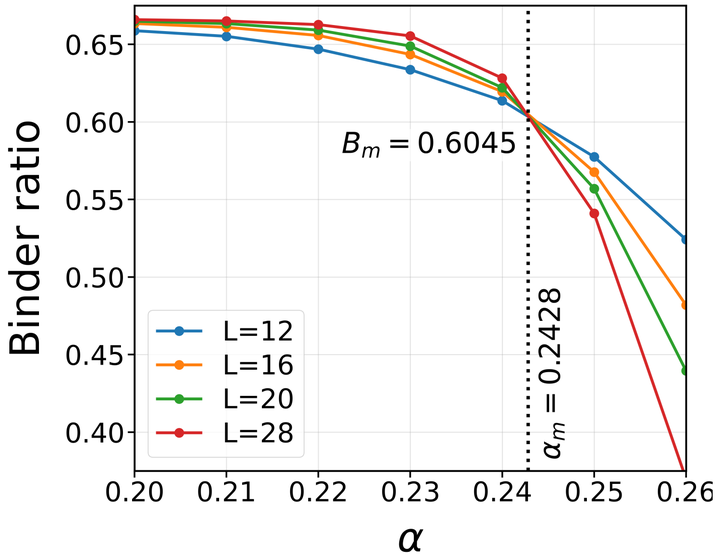
ρ_{WS}
$\propto (1 + \prod_r X_r)$
(even number of sites)

Weak Z2 symmetry: multiple transitions?

ρ_{WTSSB}
$\propto |+\rangle\langle+| + |-\rangle\langle-|$
(even number of sites)



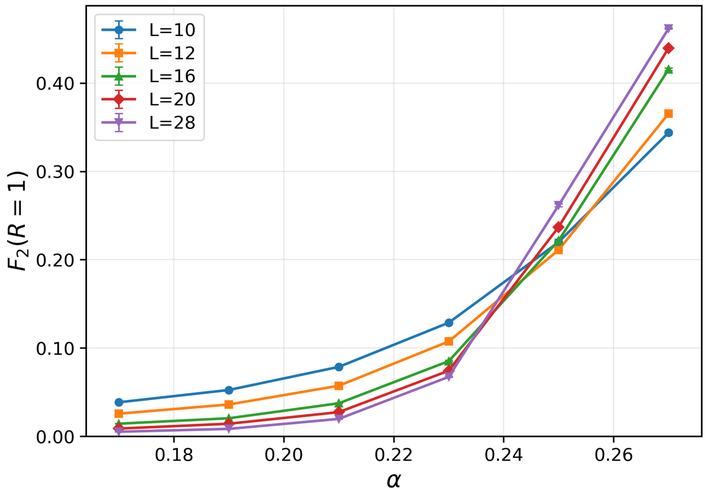
ρ_{WS}
$\propto (1 + \prod_r X_r)$
(even number of sites)

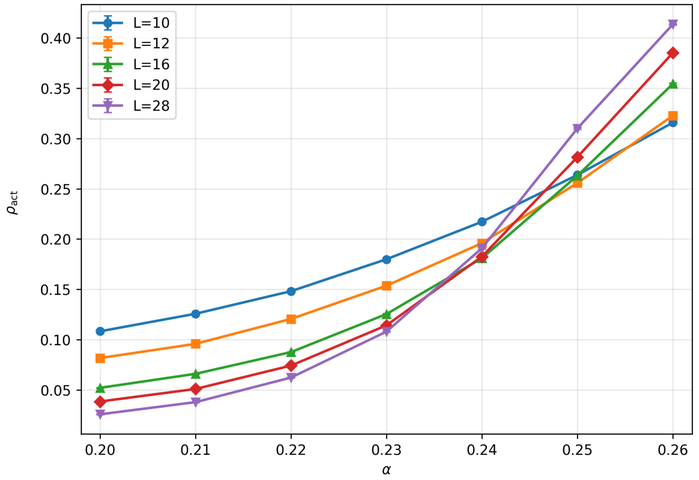


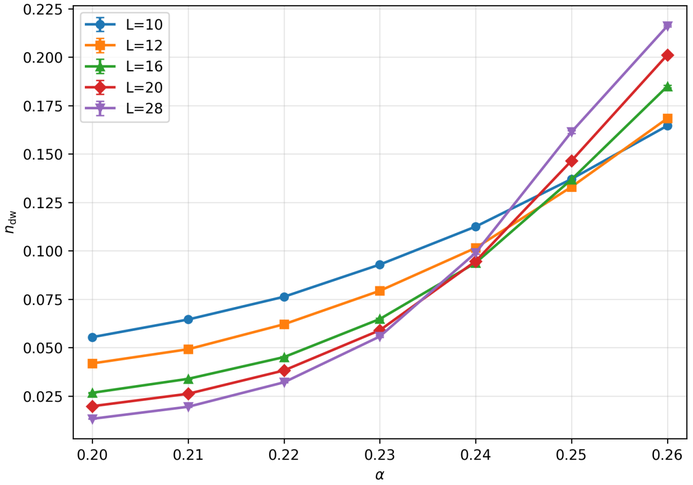
Weak symmetry transition detected by:

$$U_4 = 1 - \frac{\langle m^4 \rangle}{3\langle m^2 \rangle^2}$$

How to detect the SWSSB?







$F^R_{ij}(\rho, Z_iZ_j) =$ marginal fidelity (at radius 1)

$\rho_{\text{act}} =$ density of plaquettes not $(++++), (----)$

$n_{\text{dw}} =$ density of domain walls $(1/2L^2)\sum_{r,r'}(1 - X_rX_{r'})/2$

Cannot use n_- bc of weak $\mathbb{Z}_2$!

Summary

Absorbing state transition?

SWSSB transition!

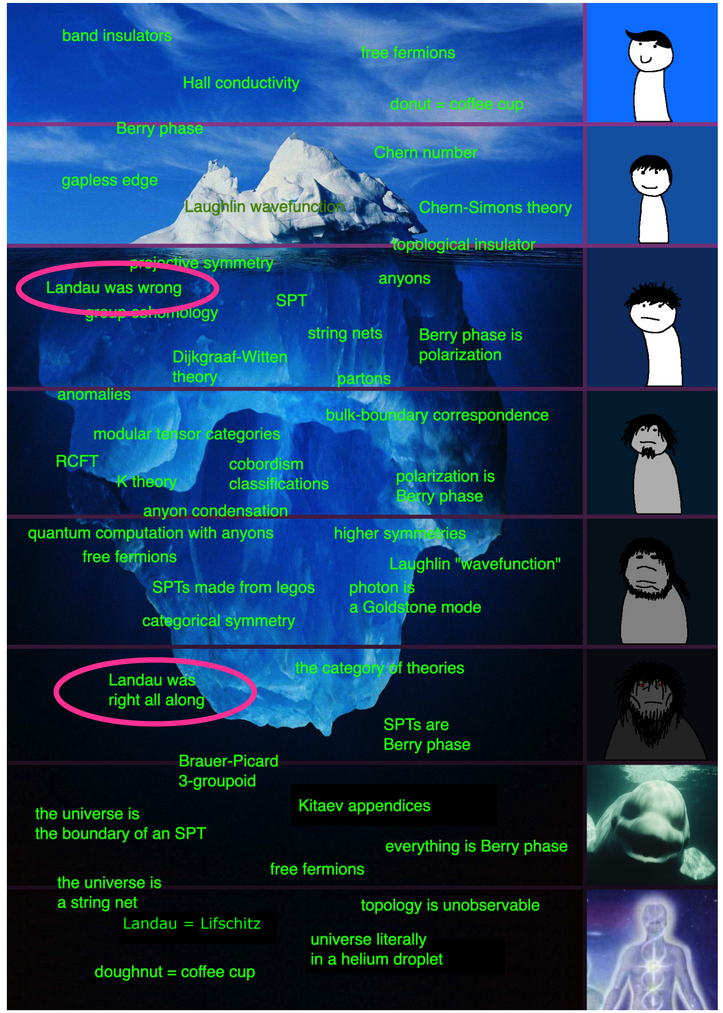


Image credit: Ryan Thorngren

PC-BARW in 1d, pair-flip Toom in 2d

Summary

Absorbing state transition?

SWSSB transition!

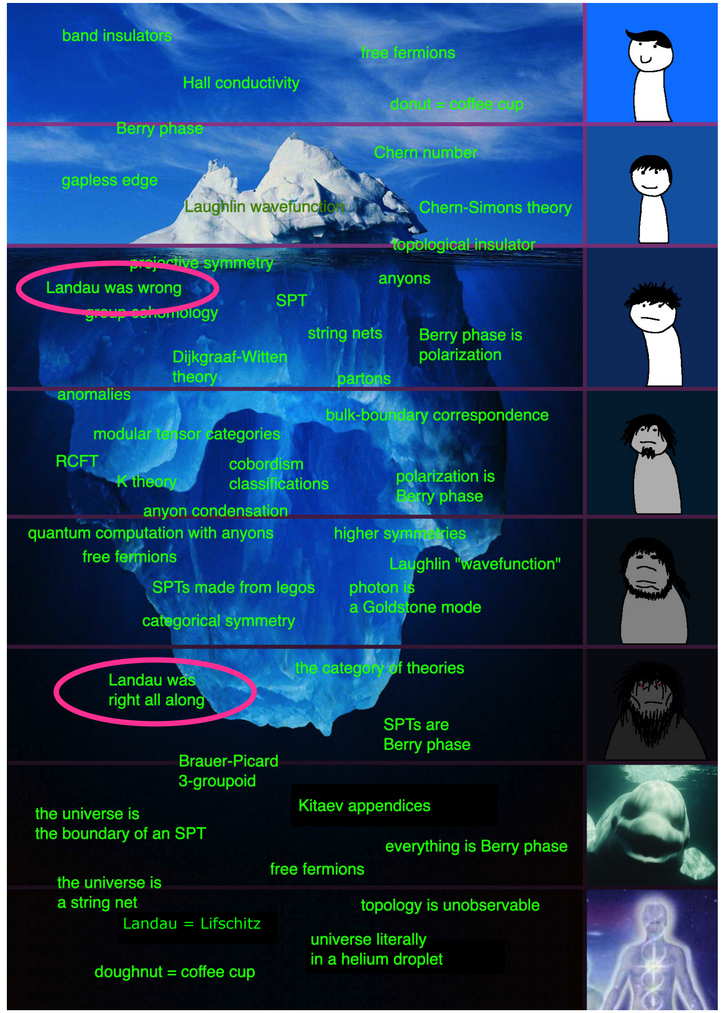
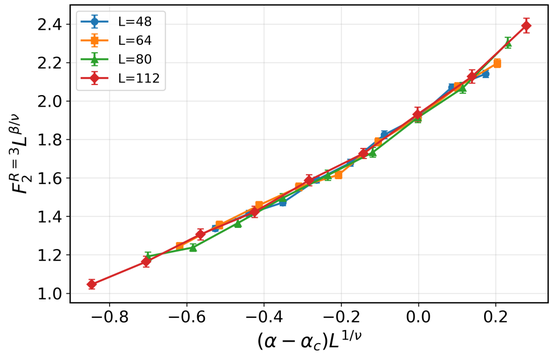
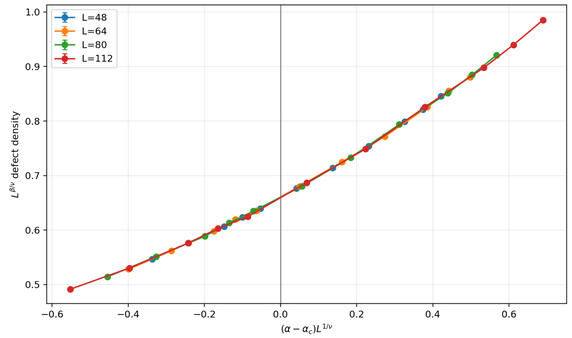


Image credit: Ryan Thorngren

Fidelity correlator



Defect density



Marginal fidelity

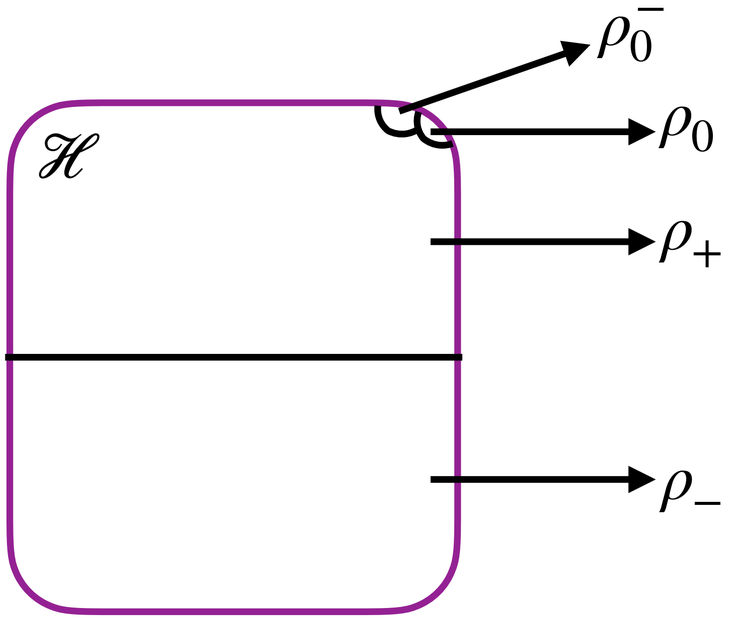
- Monotonic
- Stable
- Complexity implications
- Can lower bound in certain cases and prove convergence
- Universal

PC-BARW in 1d, pair-flip Toom in 2d

Comments/Outlook

- 2+1d model can be thought of as toric code robust to branching error.
- Rigorous proofs of stability/instability of certain steady state spaces?
- Thinking of dynamical constraints (i.e. forbidding certain branchings) in terms of the steady states they add or symmetries?
- Other kinds of phase transitions in highly non-equilibrium settings?
- More direct relationship between marginal fidelity and traditional order parameters?

Comments/Outlook



We have a decent understanding of states. But in what ways are we allowed to put them together into interesting steady state spaces? And how do we use this understanding to construct (quantum) dynamical phase transitions?